Appendix A Using the Software

The disk associated with this book contains 2 files with .exe extenders and the files that run under QBASIC that have the extenders: .bas, and .txt. The "exe" can be run from the disk by clicking on the file. Better, copy the file to your hard drive and click on it to open the file and click on the program with the .bat extender to get the startup banner ("bat" files are used in MS-DOS (Microsoft disk operating system) written in ASCII that run like .exe files). Press any key and the next page contains operating instructions. Press any key and the program starts.

The files with .bas and .txt extenders have to be run from QBASIC. The difference between them is the .txt files can be viewed with any text editor such as "Word Pad" and the files with the .bas extender require QBASIC to read them. QBASIC can save the file in either format.

BASIC was developed at Dartmouth College as a simpler to use computer code than those existing at the time. Its program language is similar to FORTRAN which was developed by IBM. It is an interpreted program in contrast with FORTRAN being a compiled program. This means the output file (.bas) from BASIC has to run using the BASIC program while a compiled file (an .exe) file will run by itself. The original BASIC was not very user-friendly from the point of editing and viewing the code. Its graphics capabilities were minimal.

Microsoft produced a vast improvement with a program sold as This retained much of the BASIC language but extended it with additional commands and an excellent graphics capability. In many ways it was better than the FORTRAN of that time (except for complex variables). QuickBASIC provides either interpreted or compiled programs. The programs are developed at interpreted programs for convenience and speed of editing. When it operates satisfactorily, the program can be compiled. While interpreted program run comparatively slowly, I have speed tested compiled versions of the "Sieve of Aristosthenes" (a prime number finder) and found speed about the same as FORTRAN, "C", or Pascal.

Windows 3, and Windows 95 contained a DOS file of useful utility programs including QBASIC which was a reduced version of QuickBASIC. The reduction being the absence of the compiling capability but it was free with the purchase of Windows. I began this book around 1990 thinking that this being free and ubiquitous is the ideal language for this book. With the passage of time, Microsoft sells VisualBASIC instead of QuickBASIC and these programs will not run in VisualBASIC. Furthermore, versions of Windows after 95 no longer have a DOS file with QBASIC. The good news is that QBASIC.exe and QBASIC.hlp can be found on the Windows CD disk in the location D:\tools\oldmsdos. An easy way to do this is to load the CD and go to Windows Explorer, click on the drive the CD is in and right click to get "find" and type in qbasic and press "enter." Load these files on your hard drive and you can run and modify any of the .bas and .txt files that are provided.

Appendix B Fundamental Physical Constants

Edited from National Institute of Science and Technology home page http://chemie.fu-berlin.de/chemistry/general/constants-en.html

Planck constant h = 6.6260755 E-34 J s
ħ = 1.05457266E-34 J s
Boltzmann constant k = 1.380658E-23 J/K
Elementary charge e = 1.60217733E-19 C
Avogadro number N_A = 6.0221367E23 particles/mol
Speed of light c = 2.99792458E8 m/s
Permeability of vacuum μ_o = 4*π E-7 T^2 m^3/J
= 12.566370614E-7 T^2 m^3/J
Permittivity of vacuum ε_o = 1 / (μ_o *c²)
= 8.854187817E-12 C2/J m
Fine structure constant α = 1 / 137.0359895
Electron rest mass m_e = 9.1093897E-31 kg
Proton rest mass m_p = 1.6726231E-27 kg
Neutron rest mass m_n = 1.6749286E-27 kg
Bohr magneton μ_B = e* h / (4*π*m_e)
= 9.2740154E-24 J/T
Nuclear magneton μ_N = e*h / (4*π*m_p)
= 5.0507866E-27 J/T
Free electron g factor g_e = 2.002319304386
Free electron gyromagnetic ratio γ_e = 2*π*g_{e*} μ_B/ h
= 1.7608592E11 1/s*T
γ_e/ (2*π) = 28.024944 GHz/T
Electron magnetic moment μ_e = -½*g_e*μ_B
= -9.2847701E-24 J/T
Proton gyromagnetic ratio γ_p = 2.67515255E8 1/s*T
γ_p/ (2*π) = 42.576375 MHz/T
Proton magnetic moment $\underline{\mu}_p$ = 1.41060761E-26 J/T
Proton-electron ratios m_p / m_e = 1836.152701
μ_e / $\underline{\mu}_p$ = 658.2106881
γ_e /γ_p = 658.2275841 (protons in water)
Charge-to-mass ratio for electron e / m_e = 1.75880E11 C/kg
Atomic mass unit AMU = 1.66057E-27 kg
Bohr radius a_o = 5.29177E-11 m
Electron radius r_e = 2.81792E-15 m
Gas constant R = N_{A*} k = 8.31451 m²*kg/s² kmol
Molar volume V_{mol} = 22.41383 m^3/kmol
Faraday constant $\mathscr{F}$ = N_A *e = 9.64846E4 C/mol
Proton g factor (Lande g factor) = 5.585

Gravitational constant G = 6.67390 E-11 $m^3/kg*s^2$
Acceleration due to gravity g = 9.80665 m/s^2
Compton wavelength of the electron λ_C = h / (m_e*c)
= 2.42631E-12 m

Appendix C Glossary

α - direction cosine angle, nuclear particle (helium nucleus), angular acceleration, fine structure constant
β - direction cosine, electron or positron, $(v/c)^2$
γ - direction cosine, electromagnetic particle, ratio of specific heats of gases, $1/\sqrt{[1-(v/c)^2]}$
δ - Dirac or Kronecker's delta function, skin depth, mass defect
ε - fast fission factor
ε_o - permittivity of vacuum
η - viscosity, number of neutrons born in fission
ξ - lethargy
θ - an angle or the azimuthal angle
ι - electrical current density
κ - heat conductivity or 1/(4*π*) for electricity
λ - wavelength, an eigenvalue
μ - reduced mass, electro-chemical potential, cosine(θ)
μ_B - Bohr magneton
μ_N - nuclear magneton
μ_o - permeability of vacuum
ν - frequency, neutrino
ρ - radius in cylindrical coordinates, resistivity, density
σ - charge density, microscopic cross section
Σ - macroscopic cross section
τ - time constant, Fermi age
ψ - wave function
$\psi^{\star}$- adjoint wave function
φ - an angle or the polar angle, magnetic flux, thermionic work function
Φ - magnetic flux, wave function, neutron flux density
Ω - solid angle, ohms (electrical resistance)
ω- angular velocity
a - acceleration
A - magnetic vector potential, atomic number, Helmholtz function
B - magnetic flus density, flexural-rigidity
c - velocity of light in vacuum, velocity of sound in air
C - electrical capacity
C_d - drag coefficient
d - deuteron
D - displacement current
e - the elemental charge
E - Energy, Young's modulus
E_o - rest mass energy
Eu - Euler's number
f - force, focal length, neutron thermal utilization

Fr - Froude's number
g - the acceleration of gravity, Lande g factor,
G - gravitational constant, shear modulus, Gibbs function
h - Planck's constant
ħ - Planck's constant divided by
H - Hamiltonian: H = T+V, enthalpy, magnetic field intensity
H_n - Hermite polynomial
i - square root of minus one, current
J - action variable, neutron current vector, total angular momentum quantum number
I - moment of inertia, sound intensity, electrical current
k - spring constant, Boltzmann's constant, wave number = $2*\pi/\lambda$
K - bulk modulus, .
L - Angular momentum, electrical inductance, Lagrangian: L = T-V, inductance, length, total orbital angular momentum quantum number
L_n - Laguerre polynomial
m - mass, magnetic quantum number, vector dipole moment
M - bending moment, force times lever arm, optical magnification
n - index of refraction, principal quantum number
N - number of, neutron number
N_A - Avogadro's number
p - momentum, object distance, neutron resonance escape probability
P - pressure, polarization, power, Poynting's vector
P_n - Legendre polynomial
Pr - probability
q - space, electrical charge, image distance, neutron slowing-down density
Q - heat, quality factor (ratio of system energy to loss per radian)
r - radial distance
R - gas constant, electrical resistance
r_B - Bohr radius
Rd - Rydberg's constant
Re - Reynold's number
s^2 - Schrödinger's number = $2*m_e/\hbar^2$
S - entropy, total spin angular momentum quantum number
t - time
T - kinetic energy, temperature
u - velocity, energy density
U - potential, internal energy,
v - velocity
V - potential, potential energy, voltage, volume
We - Weber's number
X - reactance
Y_{nm} - spherical harmonic
Z - nuclear charge, electrical impedance, proton number

$\mathscr{L}$ - Laplace tansform
$\mathscr{E}$- electric field
$\mathscr{F}$- Faraday's constant

Appendix D Answers

Chapter 1

1a. Your weight in Newtons: let X be your weight in pounds, X/2.20462 (equation 1.3-4) gives your weight in kilograms. This multiplied by the acceleration of gravity gives your weight, in units of force as: 9.8066 * X/ 2.20462 Newtons.

1b. Your height in wavelengths of red cadmium light: let H be your height in feet; your height in meters is H*12*2.54/100 (equation 1.3-3). A meter is 1,533, 164 wavelength of red cadmium light (page 1-4), so the number of wavelengths of red cadmium light in your height is H*12*2.54*1533164/100.

1c. Length of a parsec: the number os seconds in a radian is 3600*360/(2*π) = 206264. An AU is 149,501,201 km. A parsec is 206264*149501201 = 3.084E12 km..

2. Derive the Law of Cosines: $c^2 = (a+e)^2+d^2$, $b^2 = e^2+d^2$. Eliminating d gives $c^2 = (a+e)^2 +b^2 -e^2$, expanding $c^2 = a^2+2*a*e+b^2$, but e = -2*b*cosθ hence $c^2 = a^2+b^2-2*a*b*\cos\theta$.

3a. Repeat Eratosthenes calculation of the earth's diameter: the distance from Aswan to Alexandria is 5000*607= 3.035E6 ft. This is 2% of a circle. Therefore the earth circumference is 3.035E6/.02 = 1.518E8 ft.= 28,740 mi. It diameter is 28740/π = 9,147 mi.

3b. Suppose a obelisk 494.6 mi south casts a 1° shadow when the one in Alexandria casts a 7°14' shadow, what is the earth's diameter? 7°14'-1° = 6°14' = 6.233° hence 57.757 of a circle. The circumference is 28,567 mi or 9093 mi diameter.

3c. Estimate the total error if there were a 1' error in the angle measured as 7° 14' and a 10% error in the distance which was 3.035E6 ft. Let C be circumference, D be distance between Aswan and Alexandria and θ the angle of the shadow, then C = D*360/θ. The derivative is: dC = (∂C/∂D)*dD + (∂C/∂θ)*dθ = 3.035E5/ 0.02 - 3.035E6*0.002/0.02 = 1.51E7 - 3.035E5. However, we do not know in which direction the errors were made so we combine in quadrature: √(2.26E14 + 9.21E10) = 1.51E7 ft. The circumference was calculated as 1.518E8 ft. hence the percent error is 1.51E7 ft./1.518E8 ft. = 10% - the error in the distance measurement.

4. Find the time constant of the spring pendulum by dimensional analysis. The spring pendulum consists of: the mass (m), period (τ), the acceleration of gravity (g), the spring constant (k),. The dimensions are: $(M)^a*(T)^b*(L/T^2)^c*\ (M/T^2)^d = M^{(a+d)}*L^c\ T^{(b+-2*c-2*d)}$ = dimensionless. Solving: c=0, a = -d, and b=2*a = 0, b= -c, a=-2*b. If b=1, a= -½, d = ½, and τ ∝√(m/k).

5a. Divide 1+x into 1 by long division to get a series: 1=x goes into 1, 1 times. Subtract (1+x) from 1 and get -x. 1+x goes into this -x times and so on to get the series $1/(1+x) = 1 -x +x^2- x^3=$....

5b. Verify 5a using the binomial series: Let x=1 and y=x, then $(1+x)^{-1} = 1-x+x^2$..

6. Find ∫dx/(1+x) by integrating term-by-term the series obtained in 5a: $\int dx/(1+x) = x - x^2/2+ x^3/3$.

7. Demonstrate that the binomial series (1.5-43) is true using the Maclaurin series. Let $f(x) = (x+y)^n$. Differentiate with respect to y: f'(0) = n*xn , f''(0) = n*(n-1)x(n-1) .... Substituting into MacLaurin's formula gives the binomial series.

8.a. Numerically integrate sin(x) from 0 to 1. My program is:

CLS : uplim = 1

```
FOR j = 1 TO 8
nosteps = 2 ^ j: sq = 0
h = uplim / nosteps
FOR i = 1 TO nosteps
y = SIN(i * h)
sq = sq + h * y
NEXT i
LPRINT "integral by rectangular = "; sq, "No. of steps ="; nosteps
NEXT j
LPRINT 1 - COS(1)
END
```

8b. The analytical integral of sinθ is cosθ; cos(0) = 1; cos(1) = .5402, the result is .4598. The results are in order of number of steps and integral: 2,.6604; 4,.5635; 8, .5117; 16, .4858. It is expected that this way will overestimate the integral.

8c. Modification of Program 1-5 for the sine gives in order of number of steps and integral: 2, .4598; 4, .4597, 8, 45969; 16, 4596997. So Simpson's rule is quite accurate with 2 steps.

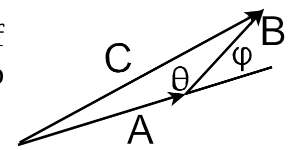

Fig. D.2-1 Vector Diagram and Angles

Chapter 2

1. Use the vector sum equation to verify the Law of Cosines and generalize it. Consider the equation $\underline{A} + \underline{B} = \underline{C}$ as diagramed in Figure D.2-1. Dot the equation on itself to get: $A^2+B^2+2*A*B*\cos\varphi = C^2$. This is not the law of cosines because a different angle is used. Notice $\varphi = \pi-\theta$ and use 1.5-18 and the parity of the sine and cosine to get: $C^2 = A^2 + B^2 - 2*A*B*\cos\theta$. To generalize, consider $\underline{A} + \underline{B} + \underline{C} = \underline{D}$. Dot this on itself to get $A^2 + B^2 + C^2 + 2*A*B*\cos\varphi_{AB} + 2*A*C*\cos\varphi_{AC} + 2*B*C*\cos\varphi_{BC} = D^2$, where φ is the angle between the vectors multiplying the cosine. In general: $R^2 = \Sigma A_i*A_j*\cos\varphi_{ij}$, where the i and js cover all vector combinations (when j = i, the angle is zero hence A_i^2).

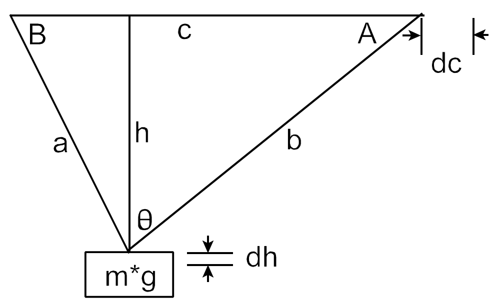

Fig. 2.10-2 Relating the Sideways Force to the Lifting Force

2. Using energy conservation, show how the force to pull a string sideways relates is related to the lifting force: Figure D.2-2 illustrates the problem, where θ is the angle opposite C etc. The force in a sideways pull is amplified. Using the Law of Cosines: $c^2 = a^2 + b^2 - 2*a*b*\cos\theta$. Differentiate w.r.t. θ: $dc/d\theta = 2*a*b*\sin\theta$. The angles of a triangle are related as: $A+B+\theta = \pi$. differentiate $d A+dB+d\theta = 0$, but $h = a*\sin B$ hence $dh = -a*\cos B*dB$. Similarly $dh = b*\cos A*dA$. From the Law of Sines, $\sin\theta/c = \sin A/a = \sin B/b$. Substituting: $dc = 2*a*c*\sin B *dh*[1/(a*\cos B)+ 1/(b*\cos A)]$. By energy conservation, $F*dh = f*dc$, where F is the weight, m*g, and f is the smaller force needed to move the support to the right. Hence
$F/f= 2*a*c*\sin B*[1/(a*\cos B)+1/(b*\cos A)]$. As $B \to \pi/2$ and $A\to 0$,
$F/f \to 2*c/A)$ hence very large.

3. A 150 lb athlete climbs 1250 ft at 100 ft/min. a) The energy is 150*1250 ft-lbs. Table

1.3-5 says 1 ft-lb = 1.356 J, hence the energy is 2.54E5 J. b) He does this in 12.5 minutes, the same table gives the conversion 150*1250/(12.5*33400) = 0.0046 hp. (It is 3.38 W). c) Apparently the muscles cannot recover energy. To the muscles, as much force is needed to lower the body as to raise it.

4. How much does the canoe move? The canoe is 14 ft long which is the distance Bill moves. The canoe weighs 90, Bill weighs 175, and Myrna weighs 120 lbs. The key thing is the center of gravity does not move. Measuring for the end where Bill was originally, and using equation 2.5-4, the center of gravity is 6 ft from this end. When Bill moves to Myrna, the center of gravity shifts to 12.36, hence the canoe moves 6.36 ft.

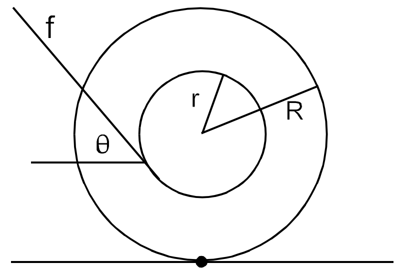

Fig. D.2-3 The Critical Angle of the Spool

5. The napkin ring has a coefficient of friction 0.3, neglect the weight of the ring for simplicity (see Jeans, 1935). If Sir Cuthbert presses with a force F, the downward force , = F*cosθ and the transverse force is f_t = F*sinθ. The coefficient of friction is $\mu = f_t / f_d = \tan\theta$. a) The critical angle is 16.7.° b) The range of angle is 33.4° c) After he passes an angle toward him of 16.7° the ring will shoot out. Whether or not it shoots out depends on the motion of his finger. If it is rigid, the ring will not get a back spin, but if it springs downward, the ring will get a spin, that will make it tend to come back. d) Whether or not he is invited back depends on his relationship with the Queen.

6a. What is the critical angle at which the spool does nothing. Figure D.2-3 illustrates the problem.. The torque balance is: f*cosθ*R = f*r, or cosθ = r/R.. For r = 1 and R = 2, the critical angle is 60.°

6b. As strings winds on, radius r increases so θ must get less.

Chapter 3

1a. What is the power expended accelerating 3000 lb to 60 mph in 8 seconds? The energy of kinetic motion is: E = ½*m*v^2 = ½*3000/32.2*88^2 = 3.607E5 ft-lb = 2.66E5 J. The power is 2.66E5/8 = 3.325E4 W = 44.58 hp (where Table 1.3-5 was used).

1b. If the engine horsepower is 225, what is the transmission efficiency? 44.58/225 = 19.8% (Assuming all of the loss is in the transmission system.)

2. What is the highway bank angle for a curve of 1000 ft radius that will through off a car at 60 mph with a coefficient of friction of 0.5? The force of friction is μ*[m*v^2/r*sinθ-m*g*cosθ]. The force pulling the car down the incline is [m*v^2/r*cosθ+m*g*sinθ.
Equating these: μ*[v^2/r*tanθ-g] = [v^2/r+g*tanθ]. Solving for the angle: tanθ = (v^2+μ*g*r)/(μ*v^2-g*r) = (7744+16100)/(3872-32200) = -.367 hence θ = 20°downward.

3. What angle cancels the centrifugal force at 55 mph for curves with a 1000 ft radius. The sideways gravitational force is m*g*sinθ. The centrifugal force is m*v^2. Equating these gives sinθ = v^2/(g*r) = 0.202; θ = 11.65°slanting downward to the inside of the curve.

4a. No, the masses and size are too great for quantum mechanics.
4b. Both momentum and energy are conserved. Momentum balance: m*v = M*V; energy balance: ½*m*v^2 = ½*M*V^2. Substituting M*V*v = M*V*V; hence v = V so m = M.

5. The colliding particle stops if the target particle has the same mass. We just demonstrated

this in problem 4b.

6. What would have been the range of the 88 if the projectile had been made of uranium (density 18.7) instead of steel (density 7.8)? We assume that the muzzle velocity is the same. The range with iron is miles. If the energy is absorbed in air friction, the range is 18.32*18.7/7.8 = 43.9 miles. However this may not be correct because the energy at the end may be different. I re-ran computer program 3-2 with the projectile mass increased to 24 kg and got 44.28 miles. So energy conservation is pretty good. (I did not try different firing angles.)

7. What is the period of a torsion pendulum using a brass disk (density 8.75) 30 cm in diameter, 1 cm thick supported by a steel wire 1mm dia. (use Table 2.7-2), 20 cm long? ; The mass is $\rho*\pi*r^2*t = 8.75*\pi*15^2*1 = 6.18$ kg; the moment of inertial is $I = ½*m*r^2 = ½ *6.18*0.15^2 = 6.95E-2$ kg-m². The stiffness (equation 2.7-37) is $k' = T/\theta = G*r_{wire}^4/(2*L)$, where G = 12E6 lb/in² = 12E6 (lb/in²)(in²/2.54² cm²)*(kg/2.2 lb) = 8.45E5 (kg/cm²)*(10²cm²/m²) = 8.45E7 kg/m². By analogy with 3.1-25, the frequency is $\omega = \sqrt{(k'/I)}$. The stiffness is k' = 8.45E7*1E-12/(2*0.2) = 2.11E-4. The angular frequency ω = 5.51E-2, the frequency is 0.17 Hz - very slow.

8. What must be the speed of the sledge before impact to just ring a gong using a 10 lb sledge to hit a 2 lb "seesaw" beam 3 ft. long, axis in the middle, to drive a 1 oz. cylinder sliding on a wire up to a gong 15 ft. above the ground (neglect friction). The velocity of the cylinder must be $½*m*v^2 = m*g*h$, or $v = \sqrt{(2*g*h)} = \sqrt{(2*32.2*15)} = 966$ ft/s. Using conversation of energy, $½*m_s*v_{si}^2 = ½*m_s*v^2 + ½*½*m_b*(L/2)^2*(2*v/L)^2 + ½*m_c*(L/2)^2*(2*v/L)^2$. Solving: $v_{si}^2 = v^2 *(10 + 2.5 + 1/16)/10$ hence the speed of the sledge is 1080 f/s. No wonder it is so hard.

9a. Haley's comet's period is 76.08 years, its eccentricity is 0.967. What is its major axis? Equation 3.12-9 is $\tau = a^{½} *\pi/\sqrt{(M*G)}$, where the sun's mass M = 1.99E30 kg, the gravitational constant G = 6.6732E-11, the period is τ = 76.08*365.25*24*3600 = 2.4E9 s. Substituting, gives its major axis a = 4.26E12 m

9b. What is the minor axis? $b = a\sqrt{(1-\varepsilon^2)}$. Substituting: b = 2.76E11 m.

9c. What is the perihelion (closest approach to the Sun)? $a = 1/[C*(1-\varepsilon^2)]$. Solving for C = 3.616E-12. $r_{min} = 1/[C*(1+\varepsilon)] = 1.40E11$ m

9d. What is the aphelion (maximum distance from the sun)? $r_{max} = 1/[C*(1-\varepsilon)] = 8.38E12$ m

9e. What is the average speed? The approximate circumference of an ellipse is $\pi*\sqrt{[(a^2+b^2)/2]}$. Substituting the circumference is 9.4832E12. The period is 2.4E9 s, hence the speed is 3.95E3 m/s = 8,830 mph.

10. Find the equation of motion of 2, 1 kg masses, constrained to move linearly and horizontally mass m_1 is connected to a wall by a mass-less spring of stiffness k = 0.001 N/m, masses m_1 and m_2 are connected with an identical spring. The Lagrangian is $L = T-V = ½*(m_1*x'^2_1 + m_2*x'^2_2)-½*[k_1*x_1^2 + k_2*(x_2-x_1)^2]$. Lagrange's equation is: $d(\partial L/\partial q'_j)/dt - \partial L/\partial q_j = 0$. The equations are: $m_1*x_1'' + k_1*x_1 - k_2*(x_2-x_1) = 0$, and $m_2*x_2'' + k_2*(x_2-x_1) = 0$. The easiest way to find the motion of the mass is with a difference equation and a computer.

11a. How much energy is required to raise 1E6 kg 500 ft. The energy is m*g*h = 1E6*9.8*500 /3.28 = 1.49E9 J.

11b. If the maximum safe peripheral speed is 100 m/s, what is the mass and rotational speed of a disk, to provide the energy required for 11a? Rotational energy $E= ½*I*\omega^2$, ω = v/r, so r*ω = 100. The moment of inertia of a disk is $I = ½*m*r^2$, hence 1.49E9 = 0.25*m*1E4, and m = 5.96E5 kg. Mass is $m = \rho*\pi*r^2*t = 7.8*\pi*0.1*r^3$. Substituting, r = 6.2 m. Since $r*\omega = 100$, $\omega = 16.1$/s, hence the frequency of rotation is 154 rpm. A real monster.

12. What is the rate of ejection of cesium that has been accelerated to a velocity v = 1.2E5 m/s to get 1 lb of thrust? Combing equations 3.3-11 and 3.3-12: f = v*dm/dt. dm/dt = 1/(0.22481*1.2E5) = 3.7E-5 kg/sec, where Table 1.3-5 is used to convert from pounds to Newtons.

13 Show that the gravitational equation is correct for a spherical mass is correct if the mass is assumed to be concentrated at the center using Legendre polynomials. The expansion of 1/s (Figure 3.9-3) in Legendre polynomials is $1/s = 1/r*\Sigma(R/r)^n *P_n(\mu)$. Using this, the gravitational potential is $V=\delta*m*G*\Sigma*\int d\varphi*d\mu *dR*R^2*\Sigma P_n(\mu)*(R/r)^n/r$ integrated over μ from -1 to 1, where $\mu = \cos\theta$. and integrated over φ from 0 to $2*\pi$. Using $P_0(\mu) =1$ and the orthogonality of the polynomials gives $\int *d\mu* P_0(\mu) *P_n(\mu) = 2$ when n = 0; But $\delta*4*\pi*R^2*dR = M$ where M is the mass of the earth. Using this, the potential of a spherical shell, hence of a solid sphere is V=m*M*G*/r. $f = -\nabla V$, so $f= m*M*G*/r^2$.

14. Correct the acceleration of gravity for the effect of the earth's rotation. Consider the earth as a sphere rotating about its axis. The resultant, using the law of cosines, is:, $geff = m*\cos\theta* \sqrt{[g^2+\cos^2\theta*(R^2*\omega^4-2*g*R*\omega^2)]}$, where θ is the latitude and R is the radius of the earth. For the earth $\omega = 2*\pi/(24*3600) = 7.27E-5/s$. The radius is 6370.95 km. $R*\omega^2 = 3.36E-2\ m/s^2$ At the north pole: $g_{eff} = g$ for which the Rubber Handbook gives g = 9.83217. Hence the formula: $g_{eff} = \sqrt{[96.671-0.65959*\cos^2\theta]}\ m/s^2$. At the equator, this gives 9.7985. The Rubber Handbook gives 9.78039. Apparently this is in error because of the non-sphericity of the earth which would decrease it at the equator.

15a. How does the gravitational potential change as you descend into a spherical earth? Consider a observer at location r inside of the earth whose outer radius is R_o. The potential for the sphere below the observer is: $V_{inner} = m*M*G/r = m*(4/3)*\pi*r^3*\rho/r. = m*(4/3)*\pi*r^2*\rho$. The potential for the earth above the observer is more difficult. In spherical geometry: $V = m*G*\rho*r^2*dr*d\varphi*d\mu/s$, where s is the distance from the observer to the differential volume. Using Legendre polynomials $1/s = (1/R)*\Sigma(r/R)^n *P_n(\mu)$. Using the trick that $1= P_o(\mu)$ and the orthogonality of the Legendre polynomials, we find: $V = 4*\pi*m*G*\rho*r*dr$. Integrating this from r to R_o gives:
$V_{outer} = 2*\pi*m*G*\rho*(Ro^2 - r^2)$. Adding the two potentials:
$V_{total} = 2*\pi*m*G*\rho*(Ro^2 - r^2/3)$.

15b. Your weight which is the force of gravity is $f = -\nabla V$. Performing the derivative gives: $f = 4*\pi*m*G*\rho* r/3$. In conclusion outside of the earth, your weight decreases quadratically with distance from the center; inside of the earth, your weight decreases linearly with distance from the center.

16a. A Foucault pendulum at the north pole rotates clockwise with respect to (w.r.t.) the earth.

16b. At the south pole it rotates counter-clockwise with respect to (w.r.t.) the earth.

16c. At the equator it does not rotate.

17a.. Show that an object with moment of inertia $I = k*m*r^2$, where k is a constant depending on the shape of the rolling object, has the same time dependence as a free falling object. Using energy conservation: $m*g*\sin\theta*s = ½*m*s'^2 + ½*k*m*s'^2$, where s is the distance down the inclined plane, and θ is the angle the plane is inclined. This has the form $A*s = s'^2$, or $dt = s'/\sqrt{(A*s)}$ which integrates (RC Handbook #131) to: $t = 2*\sqrt{(s/A)}$, or $s = t^2*g*\sin\theta/(1+k)$. The effect of the inclined plane is to reduce gravity by a factor of $\sin\theta(1+k)$.

17b. Suppose a solid sphere (k=2/5) is released to roll down a board that is 32 ft. long and inclined 5°. What is the time to reach the following distances: 2, 4, 8, 16, 32 ft? $s = 2.001t^2$, or $t = 0.7069*\sqrt{s}$. The times, respectively are: 0.999, 1.414, 2.0, 2.82, 4.0, seconds.

18a. Rearrange the longitudinal momentum so ball 2 is on the right and square the equation: $(m_1*v_1)^2 - 2*m_1^2*v_1*v_1'*\cos\theta' + (m_1*v_1')^2*\cos^2\theta = (m_2*v_2)^2*\cos^2\theta$. Doing the same to the transverse momentum: $(m_1*v_1')^2*\sin^2\theta' = (m_2*v_2)^2*\sin^2\theta$. Adding the squared equations: $(m_1*v_1)^2 - 2*m_1^2*v_1*v_1'*\cos\theta' + (m_1*v_1')^2 = (m_2*v_2)^2$. Using the energy equation to eliminate v_1': $m_1*v_1'^2 = m_2*v_2^2 - m_1*v_1^2$ gives: $v_2 = 2*m_1*v_1*\cos\theta/(m_1+m_2)$. The quantity $m_1/(m_1+m_2)$ is called the reduced mass.

18b. Ball 2 travels on a 45° angle to hit the corner pocket; use v1 = 10 f/s and the fact that the mass of the balls is the same. The speed of ball 2 after the collision is $v_2 = 10*0.707 = 7.07$ f/s.

18c. Ball 1 must hit ball 2 such that the line of centers is 45°. Using the law of cosines: $c^2 = 72^2+4 - 2*72*2*\cos(45°)$. 70.6 in. From the law of sines: $\sin\theta/2 = 0.707/70.6$, hence $\theta = 1.14°$ off of the lengthwise axis of the table.

19a. What is the efficiency of a 20 ft diameter wind-generator that generates 10 kW in a 20 mph wind? Imagine a 20 ft dia. cylinder of air. The volume of the cylinder is $\pi*d^2*v*t$. It is traveling at speed v. The power is $½*\pi*d^2*v^3 = ½*1.2*\pi/4*20^2/3.28^2*(20*5180/3600)^3 = 4.4E4$. The efficiency is 10/44 = 22.7%.

19b. We just found that the power in the wind goes as the cube of the wind speed. Hence $(30/20)^3 = x/44 = 67.5$ kW in the wind. If the efficiency remains the same, the power should be 10*3.375 = 33.7 W.

Chapter 4

1a. Given that King Hieron II's crown weighs 0.555 kg in air and 0.518 submerged in water what is the percentages of gold and silver? $W_a = \rho*V$, where ρ is the density of the crown, V is the volume of the crown, and .W_a is the weight in air. By Archimedes principle, $W_w = W_a - \rho_w*V$, where W_w. is the weight in water. Eliminating the volume and solving, $\rho = 1*555/(555-518) = 15$. x*19.32 + (1-x)*10.50 = 15, x = 0.51 or 51% gold, 49% silver.

1b. If the measurements have 1% error what is the uncertainty in the density? The logarithm of the density is $\ln(\rho) = \ln(\rho_w*W_a) - \ln(W_w - W_a)$. Differentiating, $d\rho/\rho = dW_a/W_a - d(W_w - W_a)/(W_w - W_a)$. $d\rho/\rho = 0.01 - \sqrt{(5.5^2 + 5.1^2)}/37$ or about 20%.. The square root is used for combining the uncertainties as the square root of the sum of the squares - called quadrature. The large uncertainties result of the smallness of the difference in weight in air and in water.

2. Work out a relationship between the period of the ball and γ in Ruchardt's experiment. The frequency of a spring pendulum is given by equation 3.1-25 as $2*\pi*f = \sqrt{(k/m)}$, where k is the change in force to change in length. The adiabatic perfect gas equation is $p*V^\gamma = ct$. Differentiating: $dp*V^\gamma + p*\gamma*V^{(\gamma-1)}*dV = 0$. dp = df/A, where A is the area of the tube, dV = dy*A and df is the change in force is: $df = -p*\gamma*A^2*V^{-1}*dy$. The spring constant is k = df/dy for the gas as $k = -p*\gamma*A^2/V$. The period $\tau = 2*\pi*\sqrt{(m*V)/(p*\gamma*A^2)}$. The minus sign is neglected having to do with direction.

3a. In Rinkel's method, find a relationship between the distance *L*, the ball drops and γ using other parameters from problem 2. Using energy balance, the potential energy is E = m*g*L, where L is the distance the ball drops before it makes its first rebound. The differential force, from Problem 2 is: df = - $p*\gamma*A^2*V^{-1}*dy$. The energy in compressing the gas is $\int f*dy$. f is found by integrating df: f = - $p*\gamma*A^2*V^{-1}*y$, where y is integrated from 0 to L. Energy = $p*\gamma*A^2*L^2/(2*V)$ = m*g*L. Rearranging: $\gamma = 2*m*g*V/ (p*L*A^2)$.

3b. Show that an error made in measuring *L* only affects γ linearly, while in Ruchardt's method, an error in τ affects γ to the second power. In Rinkel's method, taking the logarithm and then the derivative gives: $d\gamma/\gamma = -dL/L$. In Ruchardt's method doing the same gives $d\gamma/\gamma = -2*d\tau/\tau$. Thus the errors in Ruchardt's method are magnified by 2x but are linear in Rinkel's method.

4a. At 100°C, and a weight of 250 kg, what is the volume of the balloon? Assume that the liftoff conditions are STP. The buoyant force is the weight of air displaced: $\rho_o*V = W_g + \rho*V$, where W_g is the weight of the balloon, V is the volume of the balloon, and ρ_o is the density of STP air, and ρ is the density of hot air. From equation 4.4-15: $\rho_o*T_o = \rho*T$. Substituting and rearranging: $V = T*W_g/[\rho_o*(T-T_o)]$. At STP, ρ_o = 1.2925 kg/m^3, T_o = 288.16°K. The air temperature in the balloon is T = 388.16, hence the volume is 757 m^3, or

4b. If the balloon is spherical, what is its diameter? 11.3 m .

4c. If the balloon were filled with hydrogen, what would be the diameter? Use the equation: $\rho_o*V = W_g + \rho_h*V$. The density of hydrogen is given by equation 4.4-15 $\rho = p*M/(R*T)$. The easiest way to find it is to note that gas densities are proportional to the molecular weight. Hence $\rho_{h.}$= ρ_o*M_h/M_a = 1.2925*2.016/28.966 = 0.09 kg/m^3. Solving $V = W_g/(\rho_o - \rho_h)$ = 250/1.2025 = 207.9 m^3 and the diameter is 7.34 m.

4d. If you designed the hydrogen balloon to have 10% greater volume than the minimum, how high would it rise? Presumably the balloon is filled with hydrogen at STP and the volume remains constant hence the hydrogen pressure and density remains constant. 10% greater volume is 228.7. Using $\rho_a*V = W_g + \rho_h*V$, it rises until ρ_a satisfies this equation. Solving: $\rho_a = W_g/V + \rho_{h.}$ = 1.183. Equation 4.4-15 says that density and pressure are proportional. Therefore equation 4.5-1 becomes $\rho = \rho_o*\exp(-k*h)$ where k = 0.1185/km. Solving: $h = (1/k)*\ln(\rho_o/\rho)$. Inserting numbers, h = 0.747 km. Clearly the balloon must have more buoyancy than this to go high, furthermore the volume is not constrained hence the limp look at launch.

5a.. What force is exerted on an 18 in. dia. pipe by water traveling 10 m/s after a 90° bend? The diameter is 0.45 m, the area is 0.164 m² hence Q = 1.64 m^3/s. Equation 4.12-4 is:, Inserting numbers: f_x = 1.64E4 N. Formula 4.12-5 is: $f_y = \rho*Q*v_o*\sin\theta$; f_y = 1.64E4 N.

5b. What is the maximum force that is developed, assuming adiabatic compression, by a 10 m slug of water, a velocity of 10m/s, slamming into a 10 m empty, air-filled, blocked section of pipe. The volume of this slug is: 1.64 m^3 hence the mass is 1.64E3 kg, and its kinetic energy is: 8.2E4 J. The adiabatic law may be written $f = p_o*A*L^\gamma/y^\gamma$, where po is atmospheric pressure, L is the length of the pipe before compression and y is the length after compression. Energy is $\int f*dy = p_o*A*L^\gamma*y^{1-\gamma}$ = ½*m*v² = 8.2E4. For air, γ = 1.4, p_o = 0.1013E6 N/m^3. Solving finds y = 0.1965 m. The force is: $f = p_o*A*(L/y)^\gamma$ = 1.013E5 N or 2.27E4 lb.

6. What is the minimum flow rate that supports the card and thumbtack 1mm from the

spool? The weight of a card is 3.6E-3 lb, a thumb tack weighs 3.47E-3 lb hence both weigh 3.18E-3 kg. The hole in the spool is 8 mm in diameter and the diameter of the spool is 4 cm. Using the notation in the figure, imagine a band of area 2*π*r*dr on the face of the spool. We are interested in the force that is exerted by a pressure p(r). From Bernoulli's equation (4.8-5) $p = p_o + ½*ρ*(v^2+v_o^2)$. Equation 4.8-1 says $v/v_o = A_o/A. = A_o/(2*π*r*t)$. The differential of down force, $df_l = 2*π*r*dr*\{p_o* ½*ρ*[(v_o*A_o/ 2*π*r*t)^2 - v_o^2]\}$. Integrating from r_o to r_m, $f_d = p_o*π*(r_m^2 - r_o^2) + ½*ρ*v_o^2*(r_o^2-r_m^2) + ½*ρ*v_o^2*A_o(r_m-r_o)/t$.. The total force is $f_{tot} = -f_u +f_d + W_c = 0$. Hence: $v_o^2 = 2*W_c/ρ*[r_m^2 - r_o^2 + A_o(r_m-r_o)/t]$. The area of the 8 mm hole is 5.1E-6 m² Don't forget to convert the kilograms to Newtons to be dimensionally correct.. I get 1.2E-3 m/s, it is a gentle blow but this seems very low, but the physics seems OK.

7. Convert the one dimensional hydrodynamic equations 4.13-6 through 4.13-8 i.e : $∂ρ/∂t + v*∂ρ/∂x + ρ*∂v/∂x = 0$, $∂v/∂t + v*∂v/∂x + (1/ρ)*∂p/∂x = 0$, $∂ξ/∂t + v*∂ξ/∂x + (1/ρ)* ∂(p*v)/∂x = 0$ into three-dimensional form. Answer: $∂ρ/∂t + \underline{v}·∇ρ + ρ*∇·v = 0$, $∂\underline{v}/∂t + \underline{v}· ∇v + (1/ρ)*∇p/∂x = 0$, and $∂ξ/∂t + \underline{v}·∇ξ + (1/ρ)* ∇·(p*\underline{v}) = 0$.

Chapter 5

1. Calculate Carnot's cycle for a perfect gas starting with an adiabatic expansion. to obtain the efficiency and Clausius' theorem. Answer: step 1 begins with an adiabatic expansion from P_2 and ends at: $P_3 = P_2*(V_2 /V_3)^γ$ with a temperature of $T_3 = T_1*(V_2 /V_3)^{γ-1}$. Step 2 begins at $P_3 = n*R*T_3/V_3$ with an isothermal compression that ends at $P_4 = n*R*T_3 /V_4$. Step 3 begins at P_4 with an adiabatic compression to $P_1 = P_4*(V_4/V_1)^γ$. Step 4 begins the isothermal expansion at $P_1 = n*R*T_1/V_1$ and ends at $P_2‘= n*R*T_1/V_2$. where P_2' is the ending pressure which must equal P_2 for the cycle to close. Begin with $P_2 = P_3*(V_3/V_2)^γ$. Substitute $P_3 = n*R*T_3/V_3$, $T3 = V_4*P_4/ (n*R)$, $P_1 = P_4*(V_4/V_1)^γ$, and $P_2‘= n*R*T_1/V_2$. Equate P_2 and $P_2’$ and get $V_3/V_2 = V_4/V_1$ the ratios of volumes in a Carnot cycle. For the efficiency: $W_1 = n*R*T_1*\ln(V_2/V_1)$ and $W_3 = n*R*T_3*\ln(V_4/V_3)$, and $Q = W_1$. Efficiency is $ε = (W_3 - W_1)/Q = 1 - (T_3/T_1)$. Clausius’s theorem takes the ratio $W_1/W_3 = Q_1/Q_3 = T_1/T_3$ (using the ratio of volumes) which he generalized to $\oint đQ/T = \oint dS = 0$, for the Carnot cycle.

2. Calculate the efficiency of the Joule- Brayton cycle. Steps 1-2 and 3-4 are isentropics, steps 2-3 and 4-1 are isobarics. Answer: $p_2 = p_1*(V_1/V_2)^γ$ $p_3 = p_2$, $p_4 = p_3*(V_1/V_2)^γ$, and $p_1 = p_4$. Putting this together gives $V_1/V_4 = V_2/V_3$ - the same as for the Carnot cycle. In addition since $T_1/T_4 = (V_1/V_4)^{γ-1}$, $T_1/T_4 = T_2/T_3$. The energy is $W = Cp*(T_3 - T_2 - T_4 +T_1)$; the heat is $Q = Cp*(T_3 - T_2)$ and the efficiency is $ε = 1 - (T_1/T_2)$.

3. The tea in my teapot is 200°F. For 3/4 of a cup of tea, how much ice, at 20°F, must be added for the tea to be at 110 °F. Answer: the ice is at -6.67°C, the tea is at 93.3°C and the desired temperature is 43.3°C. The Rubber Handbook gives the specific heat of ice about this temperature as $C_p = 0.45$ cal/(gm*°C), The heat of fusion is $C_f = 79.72$ cal/gm, and the specific heat of liquid water is 1 cal/(gm*°C). To raise the ice to the melting point takes 0.45*6.67 = 3.0 cal/gm. Melting the ice takes 79.72 cal/gm, raising water from 0 to 43.3 takes 43.3 cal/gm, for a total of 126 cal/gm. To lower the tea to the desired temperature removes 50 cal/gm. 3/4 of a cup is 3/8 of a pint or 3/16 of a quart or 3/(16*1.05) = 0.178 ltr= 178 gm, hence 8900 calories are required. The amount of ice

required is 8900/126 = 70.6 gm. = 2.48 oz. of ice.

4. Find the 1^{st} and 2^{nd} TdS equations respectively: $T*dS = C_v*dT + T*(\partial p/\partial T)v*dV$, and $T*dS = C_p*dT - T*(\partial V/\partial T)p*dp$. Answer: $dS(T,V) = (\partial S/\partial T)*dT + (\partial S/\partial V)*dV$
$T*dS(T,V) = T*(\partial S/\partial T)_v*dT + T*(\partial S/\partial V)_T*dV$. But $T*(\partial S/\partial T)_v = C_v$, and $(\partial S/\partial V)_T = (\partial p/\partial T)_V$ where the last was done using the fourth line in Table 5.6-2. Putting this together gives the 1^{st} TdS equation. For the second, $dS(T,p) = (\partial S/\partial T)_p*dT + (\partial S/\partial p)_T*dp$. Multiply by T to get:
$T*dS(T,p) = T*(\partial S/\partial T)_p*dT + T*(\partial S/\partial p)_T*dp$. But $T*(\partial S/\partial T)_p = C_P$, and $(\partial S/\partial p)_T = -(\partial V/\partial T)_p$ from Table 5.6-2 line 12. Putting this together gives the 2^{nd} TdS equation.

5a.. What is the heat flux in a 1 m long heat pipe with heat conductivity κ' = 5000W/(ºC*m) with one end at 300°C and the other at 100°C? Answer: 5000*200 = 1 MW.

5b. If the heat transferring cross section is 1 cm², what is the power density in the pipe? Answer: 1MW/1 = 1MW/cm².

6. What is the temperature profile of an infinite, one-dimensional 1 m thick slab of aluminum at an equilibrium temperature of 0°C whose surface temperature is suddenly changed to 100°C? Answer: the solution for copper is calculated in Figure D.5-1. The easiest way to solve this is to change Program 5-3 for a slab of aluminum, which I did. However it is instructive to see how the parametric change affects the solution. The governing equation is $\kappa*\nabla^2 T = \partial T/\partial t$ (5.8-2) which is $\kappa\ (m^2/s)*\partial^2 T/\partial x^2 = \partial T/\partial t$ Notice that the units balance, hence reducing κ has the effect of increasing time. For copper κ = 1.2E-4 m²/s, for aluminum κ = 9.8E-5 hence time is increased by 1.22 which has the effect of relabeling the contours by 1.22 e.g. 250*1.22 = 306 sec and so forth for the other contours.

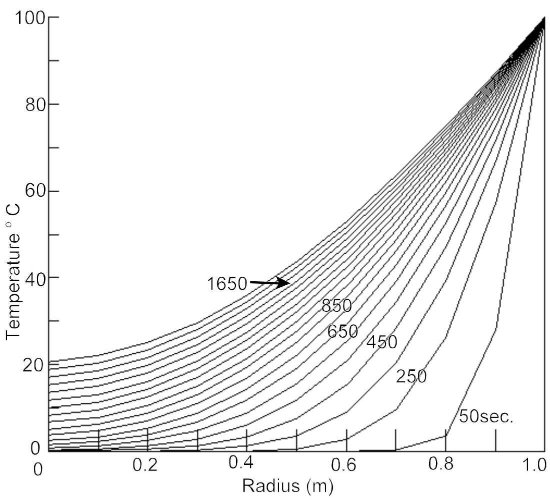

Fig. D.5-1 Transient Temperatures in a Copper Sphere

7. What is the time-dependent temperature profile of a 1 m radius copper sphere, originally uniformly at 0°C whose surface temperature is suddenly heated to 100°C? Answer: The heat conduction equation in spherical coordinates is $\kappa*(\partial^2 T/\partial r^2 + 2/r*\partial T/\partial r) = \partial T/\partial r$. Although the sphere and the slab are both one-dimensional problems, the sphere has only one surface hence only one boundary condition. I tried zero slope for the center of the sphere but it did not work. The solution was to write it as a slab going from -n to n so it would pass through zero and use the spherical Laplacian with an offset to prevent 1/r from blowing up. This is described as follows:

Program D.5-1 Transient Temperature in a Sphere

```
DIM T(50, 200)
SCREEN 11: VIEW (0, 0)-(639, 479)
WINDOW (-.2, 110)-(1.2, -10)
```

Much of this is like Program 5-3. In trying to make the index go from -n to n, the first problem you encounter is that index variable

```
4  n = 10: Tt = 190: delt = 10: m = 11
5  diff = .00012: delx = .1
6  Ct = diff * delt / delx ^ 2
7  Ct2 = 2 * diff * delt / delx
8  LINE (.1, 0)-(.1, 100)
9  LINE (.1, 0)-(1.1, 0)
10 LINE (1.1, 0)-(1.1, 100)
11 FOR i = 0 TO 100 STEP 10
12   LINE (.1, i)-(.12, i)
13 NEXT i
14 FOR i = 10 TO 100 STEP 10
15   LINE (i / 100, 0)-(i / 100, 5)
16 NEXT i
17 FOR i = -n TO n
18    T(i, 1) = 0
19 NEXT i
20  FOR J = 2 TO Tt
21    T(1, J) = 100
22    T(m + n + 1, J) = 100
23  NEXT J
24 FOR J = 1 TO Tt
25    FOR i = -n + 1 TO n
26 T(m + i, J + 1) = Ct * T(m + i - 1, J) + (1 - 2 *
27 Ct) * T(m + i, J) + Ct * T(m + i + 1, J) + Ct2
28 * (T(m + i - 1, J) - T(m + i, J)) / (i + .1)
29    NEXT i
30 NEXT J
31 FOR J = 5 TO Tt STEP 10
32   FOR i = 1 TO n
33     LINE ((i + 1) / 10, T(m + i + 1, J))-((i) / 10,
34 T(m + i, J))
35    NEXT i
36  NEXT J
37  END
```

may not assume negative values. This is overcome by biasing the variables with an offset so that they remain positive even though the x variable is going negative. For safety, line one is changed for an x dimension of 50. Lines 2-3 are unchanged; line 4 has the offset m set to 11. Line 7 is new to provide a constant for the 1/r terms in the spherical Laplacian. Lines 8-10 are changed for the graphical results as are lines 11-16. Lines 17-19 are changed for the negative space. Line 21 sets the left boundary to 100°C at x=1 corresponding to m-n where n = 10. Line 22 sets the right boundary condition to 100°C. Lines 24-30 and 25 to 29 are a double loop for stepping lines 26-28 which are the heat equation using the m offset to keep the variables positive. Lines 31-36 plot the right half of the results for r = 0 to 1 m. The results is shown as Figure D.5-1.

8. Rewrite Program D.5-1 to calculate the temperature profile of an infinite copper slab 1.0 m thick, at an equilibrium temperature of 0°C whose left face is suddenly changed to 100°C but whose right face is held at 0°C. Calculate the heat flux into the left face from a small time after the temperature transition until near equilibrium. What heat density (time integrated flux) flows during this time? Program 5-3 is modified by changing line 18 to T(N+1) = 0 to produce Figure D.5-2. The heat flux

into the left face is found by adding the lines FOR j = 3 TO tt. F = 401 * (T(2, j) - T(3, j)) / delx.
LINE (j / 200, fold / 2000)-((j + 1) / 200, F / 2000)
fold = F. NEXT j. The first line calculates the heat flux where 401 is the heat conduction κ' for equation 5.8-1. The rest of these lines plot Figure D.5-3. Notice that the heat flux approaches a constant as the distribution approaches equilibrium.

9. If a doughnut is heated, does the hole expand or contract? Answer: Suppose there is a radius R about the axis of the doughnut in the doughnut's frame of reference such that r< R, the doughnut expands inward and r > R, it expands outward.. This neutral axis radius expands because the circumference expands: $\Delta C = C_o * C_L * \Delta T$, where C_o is the circumference before heating, and C_L is the linear expansion coefficient. But $2*\pi*R_o = C_o$, hence $\Delta R = R_o * C_L * \Delta T$. The inward expansion is $\Delta r = (R_o - r_o)*C_L * \Delta T$, and $\Delta R - \Delta r =$

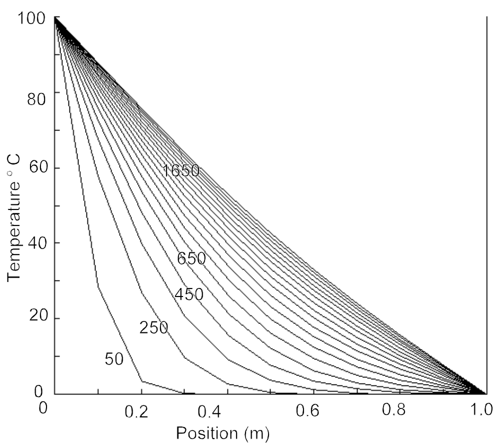

Fig. D.5-2 Temperature Profile in Copper Slab. One face at 100°C and the other 0°C.

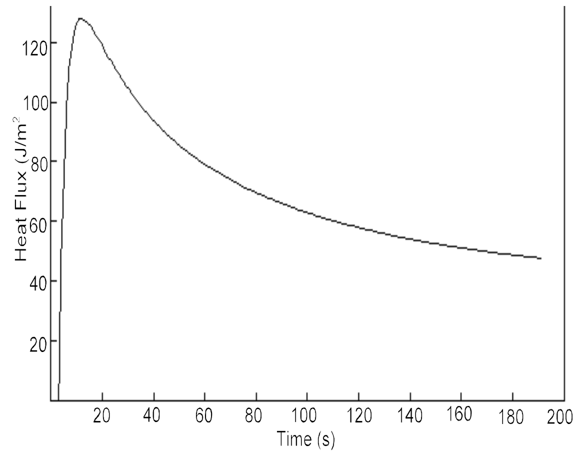

Fig. D.5-3 Heat Flux into the Slab of Figure 5.11-3.

$r_o*C_L * \Delta T$. The conclusion is that the hole expands for all conceivable doughnuts with a positive expansion coefficient.

Chapter 6

1. Show that the probability of winning at craps (2 dice) is: 244/495. Answer: From page 6-3, the probability of 7 is 6/36 and of 11 is 2/36 so the probability of a win on the first throw of the dice is 8/38.. The next part concerns making your point A with probability a, crap out B, by rolling a 7, with probability b, and C neither a 7 or my point with probability c. The probability is $P = a+c*a+c*c*a+c*c*c*a+... = a*\Sigma c^i$. a/(1-c)where the first term is for making the point on the throw after establishing the point, the second on is not crapping out but making the point on the next throw and so on. But a+b+c = 1 hence P = a/(a+b). The probability of 4 or 10 is 3/36, the probability of 7 is 6/36, hence P(4,10) = (3/36)/[(3/36)+(6/36)] = 3/9. Similarly, P(5,9) = 4/10, and P(6,8) = 5/11. The result is: 8/36 + 2*(3/9)*(3/36)+2*(4/10)*(4/36) + 2*(5/11)*(5/36) = 244/495 = 0.4929, where the 2's come from 2 possibilities (e.g. 4 or 5), and the third parenthesis is the probability of that point

to start with.

2. Show that the mean energy of a gas molecule is 3/2*k*T and the mode energy is ½*k*T. Answer: From equation 6.3-15: $<\varepsilon> = 2*\pi/(\pi*k*T)^{3/2}*\int\varepsilon^{3/2}*\exp(-\varepsilon/(k*T)*d\varepsilon$, where the denominator is normalized. and the range of integration goes from 0 to ∞. From the rubber handbook, equation 661, the integral is $\Gamma(3/2+1)*(k*T)^{3/2}/4$. $\Gamma(2+½) = 3*1*\sqrt{\pi}$. The result is $<\varepsilon> = (3/2)*k*T$. The mode $\varepsilon_m = d[\varepsilon^{1/2}*\exp(-\varepsilon/(k*T)]/d\varepsilon = 0 = ½*\varepsilon^{-½} - \varepsilon^{-½} */(k*T)$. Hence $\varepsilon_m = ½*k*T$.

3. Calculate the variance and the standard deviation. of the velocity using the M-B distribution. Answer: From problem 2, $<\varepsilon> = (3/2)*k*T = m*<v^2>/2$. Hence $<v^2> = 3*k*T/m$. From equations 6.3-19 and 6.3-22 and squaring: $<v>^2 = 8*k*T/(\pi*m)$ so VAR = 0.453*k*T/m. $\sigma = 0.673*\sqrt{(k*T/m)}$.

4. What displacement of a droplet 1.E-8 m in diameter with a specific gravity of 0.8. from a collision with a nitrogen molecule having average velocity at STP? Answer: The droplet mass is $m = 4*\pi*(0.5E-8)^3*0.8E3 = 1.25E-9$ kg.. Using equation 6.3-19, with the atomic weight of nitrogen of 14, but it forms a molecule N_2 = 28 at 273°K gives $v_p = 125*\sqrt{(273/28)}$ = 390 m/s. The mean velocity is $<v> = 2/\sqrt{(\pi)}*v_p$ = 440 m/s. The mass of a nitrogen molecule is 28*1.66E-27 kg = 4.65E-26 kg.. From equation 3.3-7, the velocity of the oil droplet after collision is v_d = 2*440*4.65E-26/1.25E-9 = 3.27E-2 m/s. The next question is how far does it go? Equation 4.11-2 says the drag force on a sphere is $f_d = 6*\pi*a*\eta*v$. The inertial force is f = m*dv/dt. equating m*dv/dt.= $6*\pi*a*\eta*v$. $s = \int v*dt = v_d*m/(6*\pi*a*\eta)$, where η = 18.4 kg/m*s. The result is s = 2.35E-11 m. This seems too small but I dn't see anything wrong..

Chapter 7

1. The frequency produced by the landing plane is 200*500/60 = 167 Hz. The velocity of sound is 331 m/s hence the wavelength is 1.98 m. The plane landing is heard according to equation 7.11-3 with λ = [1-2E5/(3600*331)]1.98 = 1.65 m. or a frequency of 200 Hz. The frequency produced by the taking-off plane is 200*2000/60 = 6.67 kHz, and the wavelength is 4.96E-2 m. The plane taking-off is heard according to equation 7.11-4 with λ = [1-3E5/(3600*331)]1.98 = 3.71E-2 m or a frequency of 8.91 kHz. The sum beat frequency is 9.12E3 Hz; the difference is 8.71E3 Hz.

2. Calculate the Fourier coefficients for a 200 Hz saw-tooth waveform. Each tooth is a right triangle with the right angle on the base, the height of the triangle is ½ of the base. From equation 7.2-3: $A_n = (1/\pi)*\int\theta/2*\cos(n*\theta)*d\theta$., and equation 7.2-4: $B_n = (1/\pi)*\int\theta/2*\sin(n*\theta)* d\theta$. By inspection, $B_0 = 0$, and $A_0 = (1/\pi)*\int\theta/2*d\theta = \pi$, when evaluated at 0 and $2*\pi$. The Rubber handbook # 393 gives $\int\theta*\cos(n*\theta)* d\theta = (1/n^2)*\cos(n*\theta) + (\theta/n)*\sin(n*\theta)$. Evaluating at 0 and $2*\pi$ gives 0. Rubber handbook # 389 gives $\int\theta*\sin(n*\theta)* d\theta = (1/n^2)*\sin(n*\theta) - (\theta/n)*\cos(n*\theta)$. Evaluating at 0 and $2*\pi$ gives $-2*\pi/n$, hence $B_n = -1/n$. The Fourier series for a sawtooth is $f(\theta) = \pi - \Sigma(1/n)*\sin(n*\theta)$ or $f(t) = \pi - \Sigma(1/n)*\sin(n*1250*t)$

3a. Compute the inverse Fourier transform of the Dirac delta function.. Equation 7.3-6 becomes, $a(t) = 1/\sqrt{(2*\pi)}*\int\delta(\omega_0)*\exp(i*\omega*t)*d\omega = 1/\sqrt{(2*\pi)}* \exp(i*\omega_0*t)$ where the integral is from -∞ to ∞.

3b. Show that the Gaussian, $f(x) = \exp(-x^2/2)$ is its own Fourier transform. Using the same equation a(k) = 1/ √(2*π)*∫exp(-x^2/2)*[cos(k*x) - i*sin(k*x)]*dx. But notice that f(x) is even and the cosine is even thus the integration can be from 0 to ∞ for the cosine and double it, while the sine, which is off cancels. The Rubber Handbook #679 is ∫exp(-a^2*x^2)*cos(b*x)dx = √(π)/(2*a)*exp[-b^2/(4*a^2)] for integration from 0 to ∞. Let a = 1/√(2) and b = k, and we get. Doubling this for -∞ to 0 gives a(k) = exp(-k^2/2). Hence the Gaussian is its own Fourier transform.

4a. What is the fundamental resonant frequency of a steel wire 0.5 mm in diameter, 0.5 m long wire under a tension of 10 N? The density of steel is 7.87E3 kg/$m^{3.}$ The area of the wire is π*0.25/4 = 0.196 /1E6 = 1.96E-7 m^2, and the volume of a meter is 1.96E-7 m^3. Multiplying by the density 1.96E-7 * 7.87E3 = 1.54E-3 kg/m. Equation 7.5-7 gives: ν = √(10/1.54E-3)/(2*0.5) = 80.5 Hz.

4b. The wire of 4a is closely wrapped with uranium wire 0.3 mm in diameter. What is the frequency of the wire? The density of uranium is 19E3 kg/m^3. The area of the wire is π*0.09/4 = 0.0707 /1E6 = 7.06E-8 m^2. The circumference wrapped on the wire is π*(0.25+(0.15) = 1.26 E-3m, so one circle has a volume = 7.06E-8 *1.26 E-3 = 8.9E-11. And a mass: 8.9E-11.*19E3 1.69E-6 kg. Each meter has 1/0.3E-3 = 3.33E3 rings. Hence the mass per meter is 1.69E-6 *3.33E3 kg = 5.63 E-3 kg/m and a total mass/m = 7.17E-3.Equation 7.5-7 gives: ν = √(10/7.17E-3)/(2*0.5) = 37.3 Hz.

5. A patient running 3 km/h and screaming in C above middle C is chased by his athletic dentist running 5 km/h. What is the tone, the dentist hears? Table 7.10-1 says C above middle C is 1046.4 Hz, or a wavelength 331/1046.4 = 0.316 m. The relative difference in speed is 2 km/h. Equation 7.11-4 gives λ = [1-(2/331)]*0.316 = 0.314, or a frequency 1051 Hz.

6. What is the fundamental resonance frequency of a rectangular parallelopiped box having dimensions: 1, 0.5, 0.25 m? In the discussion in Section 7.7, it is obvious that the change from a two-dimensional geometry to a three-dimensional: ν = c/2*[$(i/a)^2$ + $(j/b)^2$ + $(k/c)^2$]. The lowest frequency occurs for the eigenvalues, 1,0,0 gives: ν = c/2*[$(1/1)^2$ + $(0/.5)^2$ + $(0/.25)^2$] = 165.5 Hz.

7. What is the motion described by equation 7.9-1, ignoring the damping term, if the driving term, f(t), is a 0.25 m step function? The mass is 10 kg and the spring constant is 10 N/m. ω = √(k/m) = √(10/10) = 1. Equation 7.9-13 becomes

$$y(t) = a*\omega*\int_0^t \sin[\omega*(t-t_0)]*dt_0 = 0.25*[1-\cos(t)], \text{ for } t>0.$$

8. Figure 7.14-1 shows three bars from Beethoven "Ode to Joy" from his 9th Symphony based on a poem by Schiller. Decode its notes and write a BASIC computer program to play your solution. The notes are bcddcbaggab, all quarter notes except the first which is a half note. The program is:

```
' Ode to Joy
DATA B,C,D,D,C,B,A,G,G,A,B
temp = 3
FOR J = 1 TO 11
READ note$
SELECT CASE note$
END SELECT
dur = 1
SELECT CASE J
      CASE 1: dur = 2
END SELECT
SOUND freq, temp * dur
```

```
CASE "A": freq = 440
CASE "B": freq = 493.82
CASE "C": freq = 523.19
CASE "D": freq = 587.25
CASE "E": freq = 659.17
CASE "G": freq = 391.94
SOUND 0, temp
NEXT J
END
```

The explanation is similar to that for Program 7-12.

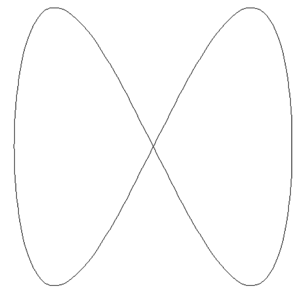

Fig. D.7-1 Lissajous Figure for the y-axis having twice the frequency of the x-axis.

9. What is the Helmholtz resonance if the car's interior has a volume V = 229 cm x 122 cm x 81 cm. = 2.263 m^3; the choke is: S = 2 cm by 45.8 cm. = 9.16E-3 m^2, and the length of the choke is L = 0.01 m. The effective length of the choke is (7.4-11): L_e = 0.01+0.8*√(9.16E-3) = 7.28E-2 m. Equation 7.4-13 is: ν = 331/6.28*√[9.16E-3/(2.263*7.28E-2) = 12.4 Hz.

Jules Antoine Lissajous (Lees-a-zhoo, 1822-1880) a professor at the Lycée Saint-Louis became interested in visually demonstrating vibration which he did by mounting mirrors on tuning forks. A beam of light was reflected from two mirrors turned at right angle to each other. If the x mirror is rapidly spinning to give a linear time base and the y mirror is mounted on a tuning fork, a sine wave is seen on the screen. If both the x and y mirrors are executing sinusoidal motion the projection is called a Lissajous figure. (Nowadays, these are easily demonstrated with an oscilloscope for plotting the x and y pattern electronically).

10a. Show that if x = sin(ω*t) and y = sin(ω*t), the pattern is a straight line. The plot is y = x, a line inclined at 45°.

10b. Show that if x = cos(ω*t) and y = sin(ω*t), i.e. 90° out of phase, the pattern is a circle. The plot is $y^2 + x^2 = 1$ which is a circle with radius 1.

c) If x = sin(ω*t) and y = sin(ω*t+δ), the pattern is an ellipse. What is the major and minor semi-axes? Since both signals have equal amplitude, the pattern is slanted at 45°. To put it in the standard position, rotate it clockwise so the major axis is on the x axis. this is done with the rotation matrix for -45° giving √(2)*x = sin(θ) +sin(θ+δ) and √(2)*y = sin(θ) -sin(θ+δ) (changing x' to x and ω*t to θ). x_{max} occurs then θ = 0 thus x_{max} = sin(δ)/√(2). y_{max} occurs when θ = 90° hence y_{max} = [1-cos(δ)]/√(2). Thus the pattern is: $x^2/\sin^2(\delta) + y^2/[1-\cos(\delta)]^2 = ½$. Notice that both the semi-major and semi-minor axes.

10d. If one wave has twice the frequency of the other what will you see? It looks a twisted loop. A simple computer program is written to plot it (Figure D.7-1).

Chapter 8

1. Römer used the eclipses of the Moon of Jupiter as a light "chopper" to measure the velocity of light. The position seen from earth at its closest approach to Jupiter is $\sin[\omega^*t_1+k^*(r_j-r_e)]$, where ω is the angular speed of a moon, r_j is the radius of Jupiter's orbit about the sun, and r_e is the earth's radius about the sun. The position when earth is farthest from Jupiter is $\sin[\omega^*t_2+ k^*(r_j+r_e)]$ Assume that the measurements are made when ω^*t_1 and ω^*t_2 are multiples of π then the time

difference between phase angles is $\omega*\Delta t = 2*k*r_e$. But $k = 2*\pi/\lambda = 2*\pi*r_e/c$. This time difference is 986 s. The earth's radius from the sun is 1.49E11 m. What is the velocity of light by this way of measurement? $\Delta t = 2*r/c$, $c = 2*r/\Delta t$ = 2*1.49E11/986 = 3.02E8 m/s.

2. What rotational speeds of the disk pass the reflected light in Fizeau's experiment? The distance from the chopper to the mirror was m; the chopper had 720 teeth. Light travels both distances in t = 2*8933/3E8 = 5.95 E-6. The angular distance between teeth is $\theta = 2*\pi/720 = 2*\pi*f*t$. f = 1/(720*5.95E-6) = 233 rps = 1.4E4 rpm. Pretty fast!

3. What glass is needed to make an achromatic triplet lens having a focal length of 4.39 cm consisting of a central lens of high crown glass, focal length 10 cm combined with two converging lenses. Using equation 8.6-8: 1/10 + 2/f = 1/4.39, f = 15.67 cm. Equation 8.6-17 gives: δ = 0.0192*15.67/10 = 0.03 so heavy flint should do the trick.

4. Some mechanisms that cause line broadening are: resolution of spectrometers, thermal agitation of the emitting atoms, Doppler effect, and the Heisenberg Uncertainty Principle.

5. Design a 100 power telescope, 0.5 m long The ratio of focal lengths is the magnification. M = F/f. The length L of the telescope is the sum of the length of the object lens and the eyepiece: L = f + F. Substituting M = F/(L - F), and F = M*L/(M + !) = 50/101 = 0.495 m. The eyepiece has a focal length 100 = .495/f or f = 0.00495.

6. Design a microscope having a magnification of 1000. Manufacturers usually make the tube length L = 18 cm. Equation 8.6-11 is: M = (F/L)*[(25/f)+1]. If we make the objective magnification = 100, the eyepiece has a magnification of 10 = F/L, then F = 180 cm. 100 = (25/f) +1, then f = 0.25 cm.

7a. What is the emitting power of the sun with a surface temperature is about 6300°K, its diameter is 8.654E5 miles = 1.393E9 m. Assume it is a blackbody radiator. Equation 8.11-5 is: I` =` $\varepsilon*\sigma*T^{4}$.. The power emitted is $I*A = 1*5.67E-8*1.575E15*\pi*1.94E18/4$ = 1.356E26 W. It candea (equation 8.11-6) is: cd = 1.6528*1.356E26 = 2.24E26.

7b. The solar constant, used for designing solar panels is approximately 1 kW/m² at high noon. Is this consistent with your radiating power calculation? The distance the earth is from the sun is 1.5E11 m. The power on 1 m² is $1.356E26 W*1/(4*\pi*2.25E22)$ = 479 W. It appears that we have over-estimated the emitting power and/or the attenuation of the radiation as well as the latitude of the solar panel.

8. The vertical length of the mirror must be 3 ft. If the line of sight is horizontal, the hat is seen; if the line of sight is 45° the shoes are seen. Since the image is a far from the mirror as the object, the mirror vertical length is 3 ft. (This is lightly in error since your hat is heigher than your eyes.

9. A radiometer consists of four 1 cm² squares, in partial vacuum, mounted so that the center of each square is 1 cm from the axis of rotation. The squares are black on one side and shiny on the other. Assume the emissivity of the black side is 1 and the shiny side is 0, what is the stalled torque when the radiometer is exposed to light at solar constant intensity. Only one square is exposed to the sun hence the energy falling on it is: 1 kW/m²*1E-4 m²/cm² = 0.1 W. Assuming this energy goes into changing the velocity of the gas molecules: $\Delta E = ½*m*(v_b^2 - v_a^2) = m*v*\Delta v = v*\Delta p$, where v

is the average velocity. But $\Delta p = f*t$; power is: $P = E/t$. $\Delta E =$ m*v* so P = v*f. From equation 4.5-4: $v_{rms} = \sqrt{(3*R*T/M)} = \sqrt{(3*8.314*273/14)} = 486$ m/s, where T = 273, R = 8.314, and M =14 (nitrogen). So f = 0.1/486 = 2E-4 N and the torque is 2E-4 N *0.01 = 2E-6 N*m.

10a. What is the focal length of a concave mirror having a magnification of 0.2 for an object 1 m. away? The magnification is M = q/p; 0.2 = q/1; the mirror equation is: 1 + 5 = 1/f. so the focal length is 0.167 m.

10b. What is the focal length of a convex mirror having a magnification of 0.2 for an object 1 m. away? From the above, q = 0.2; the mirror equation is 1 - 0.5 = 1/f. f = -2.5 m.

11. What is the focal length a gas acoustic concave lens having 10 m radius of both faces. The gas of the lens is at 373°K; the outside air is 273°K. Using equation 4.13-23 the index of refraction is $n = \sqrt{373/273} = 1.168$. Equation 8.6-3 gives the focal length as 1/f = 0.168 * (1/10 + 1/10) = 29.76 m. There is nothing in equation 4.13-22 that is frequency dependent therefore the lens is acromatic.

12. What rotational speed of a 16 spokes wheel looks stationary when photographed with a 20 times per second camera? What speed appears to be going backward? What speed appears to be going forward? The angle between spokes is $2*\pi/16 = 2*\pi*f/20$. f = 1.25 rps = 75 rpm. For a 3 ft wheel this is 8.03 mph. If the wheel is rotating faster, it appears to be going forward; if it is rotating slower, it appears to be going backward.

13. What is the diameter of a hair under one edge of optically flat plates showing 60 dark fringes under 0.5 μm wavelength light. Two adjacent fringes differ by a wavelength. Therefore the hair is 30 μm in diameter uncertain by about 0.5 μm.

Chapter 9

1a. Show that the angular oscillation frequency of a compass needle with magnetic moment m and moment of inertia I_m in a magnetic field H is: $\omega = \sqrt{(m*H/I_m)}$. Answer: The period of a torsion pendulum is: $\omega = \sqrt{(I_m/k)}$ where I_m is the moment of inertia and k is torque per angle or torsion stiffness (equation 3.1-25). The torque on a magnetic dipole is $\underline{k} = \underline{m}x\underline{H}$ (equation 9.5-10). Substitute and it is shown.

1b. In an Earth's field of 2E-5 Tesla (T), if the compass needle has a moment of inertia is 1E-7 kg*m², the needle is 3 cm long, and the pole strength is 5E-3 Nm²/T, what is the period of oscillation of the needle ? Answer: $\tau = 2*\pi*\sqrt{(m*H/I_m)} = 2*\pi*\sqrt{(2*0.03*5E-3*2E-5/1E-7)} = 0.38$ s.

2a. What resistance is measured across one resistor in an infinite triangular symmetrical two-dimensional mesh of 1Ω resistors? Answer: If we draw the triangular mesh, we note there are 6 resisters connected to each node. Inject 1 A into a node and 1/6 A flows in a resistor producing 1/6 V; a 1 A sink, causes 1/6 V. Since 1 A is being supplied, the resistance is 1/3 Ω.

2b. What resistance is measured across one resistor in an infinite cubic symmetrical three-dimensional mesh of 1Ω resistors? Each node has 6 resistors connected to it, so the answer is the same as 2a - 1/3 Ω

2c. What resistance is measured across two connected in-line, resistors in an infinite square symmetrical mesh of 1Ω resistors? Call the ends of the two resistors node 1 and node 2. If a current source is connected to node 1, 1/4 A flows in a resistor connected to the node. This current flows to the second resistor and splits 3 ways at node 2 with 1/12 A in each branch. The voltage between node 1 and 2 is 1/3 V. Remove the source and connect a sink at node 2. The voltage on the resistor connected to node 2 is 1/4 volt and the one connected to node 1 is 1/12. Now connect the source and sink at the same time and the voltage is across both resistors is 2/3 V with 1 A being supplied, the resistance is 2/3 Ω.

3a. Prove the vector identity : $\underline{A}x(\underline{B}x\underline{C}) = (\underline{A}\cdot\underline{C})*\underline{B} - (\underline{A}\cdot\underline{B})*\underline{C}$. Answer: Using equation 2.2-19, BxC = $i*(B_y*C_z-B_z*C_y)+j*(B_z*C_x-B_x*C_z)+k*(B_x*C_y-B_y*C_x)$, and Ax(BxC) = $i*[A_y*(B_x*C_y-B_y*C_x)-A_z*(B_z*C_x-B_x*C_z)]-j*[A_x*(B_x*C_y-B_y*C_x)-A_z*(B_y*C_z-B_z*C_y)+k*[A_x*(B_z*C_x-B_x*C_z)-A_y*(B_y*C_z-B_z*C_y)]$. Grouping by vectors, this is: $i*[B_x*(A\cdot C -A_x*C_x)]+j*[B_y*(A\cdot C -A_y*C_y)]+k*[B_z*(A\cdot C -A_z*C_z)] -i*[C_x*(A\cdot B -A_x*B_x)]-j*[C_y*(A\cdot B -A_y*B_y)]-k*[C_z*(A\cdot B -A_z*C_z)]$. Cancellations take place and it is $(\underline{A}\cdot\underline{C})*\underline{B} - (\underline{A}\cdot\underline{B})*\underline{C}$.

3b. Prove: $\nabla\cdot\nabla x\underline{A} = 0$. Answer: The $\nabla x\underline{A} = i*(\partial A_z/\partial y-\partial A_y/\partial z) +j*(\partial A_x/\partial z-\partial A_z/\partial x)+ k (\partial A_y/\partial x-\partial A_x/\partial y)$. The x divergence is zero because there is no x component, and similarly for y and z.

3c. Prove: $\nabla x(s*\underline{A}) = s*\nabla x\underline{A} + (\nabla s)x\underline{A}$.. Answer: $\nabla x(s*\underline{A}) = i*(\partial s*A_z/\partial y-\partial s*A_y/\partial z) +j*(\partial s*A_x/\partial z- \partial s*A_z/\partial x)+ k (\partial s*A_y/\partial x-\partial s*A_x/\partial y)$. Using equation 1.5-43, the s is pulled out and we get $s*\nabla x\underline{A}$. With the derivative action on s, we get ∇s, but the vector is caught in the curl so $(\nabla s)x\underline{A}$.

4. Two rectangular 1 m loops each carrying 1 A of current are placed such that the sides immediately above each other are parallel. What is the force that one loop exerts on the other? Answer: This is essentially the problem of 4 times the force of 2 1m long parallel wires spaced 1 cm apart. The right angle ends are orthogonal so the answer is approximately given by equation 9.6-16. F = 4*4*π*1E-7 /(2*π*0.01) = 8E-5 N.

5a. A large magnet has pole tips of 0.5 m² area, gap of 0.1 m and is uniformly filled with 2 T magnetic flux density. What energy is stored in the gap? Answer: Equation 9.5-19 says the energy U = 0.5*4*0.5*0.1/ 4*π*1E-7 = 8E4 J.

5b. What is the force tending to pull the poles together. Answer: U = p*V, the volume is 0.05 m^3 hence the pressure is 8E4/0.05 = 1.6E6 N/m². The area is 0.5 m², so the force is 8E5 N.

6a. A large gas dielectric capacitor has plates of area 0.5 m², a gap of 0.01 m and is charged to a voltage of 100,000 V. What energy is stored? The electric field is 1E5/0.01 m = 1E7 V/m. Using equation 9.3-19, U = 0.5*8.854E-12*1E14*0.5*0.01 = 2.21 J.

6b. What is the force tending to pull the plates together? Using U = p*V, the pressure is 2.21/(.5*0.01) = 443 N/m².

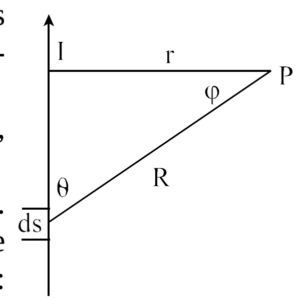

Fig. D.9-1 Self-Inductance

7a. The field of a flat coil is axial only in a small volume on axis. However two parallel axial equal coils has a larger region. Find the optimum spacing. This device is called a "Helmholtz coil." Answer: equation 9.6-8 for the magnetic field of a current loop is $H = I*a^2/[2*(a^2+z^2)^{3/2}]$. Two parallel axial coils at z and -z have a field H =

$I*a^2/2*\{1/[a^2+(L+dz)^2]^{3/2}+1/[a^2+(L-dz)^2]^{3/2}\}$, where one coil is at L from the origin and the other is at -L, and dz is a perturbation from the origin. Ignoring dz^2 and setting L = a, we get $H = I*a^2/[2*\sqrt{}(2*a)]$ and find the variation in H is second order if L = a..

7b. If each coil has a radius of 30 cm, 100 A-turns, what is the flux at the center? Answer: $H = 100*0.3^2/[2*\sqrt{}(2*0.3)] = 5.8$ webers.

8. Compute the self inductance of a straight wire (Figure D.9-1). Answer: Using the sketch to the right, equation 9.6-1 is: $dH = I*\sin(\theta)*ds/(4*\pi*R^2)$. Changing angles and noting that $R = r*\sec(\theta)$, $ds = r*d\varphi/\cos^2(\varphi)$ to get: $H = [I/(4*\pi*r)]*\int\cos(\varphi)d\varphi = I/(2*\pi*r)$ which is the magnetic field of a straight wire. In computing self inductance, it is reasonable to treat the inside of the wire separately from the outside. We assume that outside, the field is the same as if the current were concentrated at the center of the wire - like the gravitational field inside of the earth hence $H = I*r/(2*\pi*a^2)$ Equation 9.5-19 says: $U = \int \underline{B}\cdot\underline{H}*d\tau = \int \mu_o\cdot H^2*d\tau = \mu_o*I^2/(2*\pi*a^2)^2*\ell*\int r^2*2*\pi*r*dr = \mu_o*I^2*\ell*/(16*\pi)*$ but $U = ½*L*I^2$. Therefore, the self inductance per unit length is: $L = \mu_o/(8*\pi)$.

9. Using the approximation that all of the current is flowing in the skin depth about a wire, what is the resistance of a 1 mm dia.10 cm length of copper wire at a frequency of 30 Mhz. Answer: Using equation 9.10-28, the skin depth at 30 Mhz is $\delta = 1/\sqrt{}(4*\pi*1E-7*5.81E5*\pi*3E7) = 0.12$ mm. The area of the skin depth is $\pi*(0.5^2 - 0.38^2) = 0.33\ mm^2$. From equation 9.4-7, the resistance/m = 1.72E-6 /(0.33*1E-6) = 5.2 Ω

10. Show that radiation contours (Poynting's vector) from a dipole antenna are circular in the r,φ plane. Answer: A dipole has circular symmetry in the plane to which the axis of the dipole is perpendicular. This was implicit in the derivation.(section 9.10-5).

11. What is the resistance of shunt resistances each of which goes like the Fibonacci series: 1,1,2,3,5,8,13,21,...? Answer: The terms in the Fibonacci series are: f(n) = f(n-1) + f(n-1), where f(1) = 1 and f(2) = 1. The easiest way to sum these shunt resistors is with a computer program

```
1 CLS: mho = 0
2 FOR n = 3 TO 100
3 mho = mho + 2 + 1 / (2*n-3): r = 1 / mho
4 NEXT n
5 PRINT r
```

Line 1 clears the screen, and zeros the accumulator for the reciprocal resistance. Lines 2-4 that computes the parallel resistances as it generates the values from the Fibonacci numbers. It takes the reciprocal to get the resistance. and sum their reciprocals. The answer after 99 terms is 5.11E-3, and after 100 terms it is 5.0688E-3. The convergence is very poor. In fact after 100,000 terms it is 5E-6 Ω and it is assumed to go to zero with an infinite number of terms

12. Use the addition theorem for the associated Legendre polynomials to show that the *cos(ψ)* between two unit vectors one of which is at the polar angle *θ* and azimuthal angle *φ* and the other unit vector having the angles *θ'* and *φ'* have the result that: $\cos(\psi) = \cos(\theta)*\cos(\theta') + \sin(\theta)*\sin(\theta')*\cos(\varphi-\varphi')$. Using equation 9.2-92 with n = 1, m = 1, $P_n[\cos(\theta)] = \cos(\theta)$. Using Rodrigue's formula (9.2-93), $P_{1,1}[\cos(\theta)] = \sin(\theta)$. Making these substitutions, it is proven.

13. If $\mathscr{E}$ and *H* are 90° out of phase with each other why is this not a case of "wattless power" and no power can be transmitted by electromagnetic waves? AC power is given by $p = I*V*\cos(\theta)$,

where θ is the phase angle. However, it is $\mathscr{E}$ and H that are 90° out of phase. Now I and H are in phase, but $\mathscr{E} = \nabla V$, if $\mathscr{E}$ is a sinusoid, its derivative is 90° phase shifted hence V and I are in phase. and power is produced.

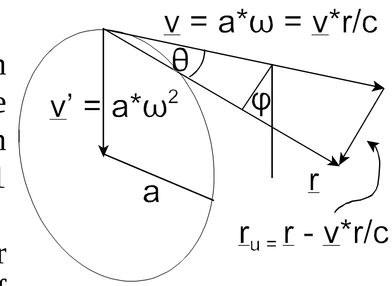

Fig. D.10-1 Diagram for Synchrotron Radiation

14. How long does it take for a signal to travel 1 m in RG8A/U cable? Answer: The impedance is 52 Ω and the capacitance is 29.5. According to equation 9.9-23, the propagation time τ = 52*29.5 = 1.53 ns/ft. The time to travel a meter is: 5 ns. (1 ns = 1E-9 s).

15. What is the capacity per unit length of a 1 mm diameter wire spaced 1 cm in each perpendicular direction from the inside of a 90° angle-shaped conductor in air. Answer: Using images in a cylindrical geometry, referring to Figure 9.2-11 upper right, and using equation 9.2-38, the potential is V = 2*κ*σ*[ln(1/ρ -2ln(1/2*a)+ln[2*a*√(2)]. The capacitance is C = 1/{2*κ*ln[a/(ρ*√2)]}, where a is the perpendicular distance to the nearest side and ρ is the radius of the wire. Hence C = 1/{2*9E9*ln[0.01/(.0005*1.414)]} = 21 pf/m.

16. Suppose a spherical conductor 1 cm in radius is centered inside a 10 cm radius conducting spherical shell with the space between filled with carbon. What is the resistance between the spheres? Answer: The voltage about a sphere is the same as the voltage about a point charge: V = q*κ/r (9.1-10). The voltage difference between two concentric spheres is V_1-V_2 = q*κ*(1/r_1-1/r_2). q = C*V, hence C = r_1*r_2/[κ*(r_1-r_2)] From 9.4-23, the resistance is R = R_v*(r_1-r_2)/(4*π* r_1*r_2), where R_v is the resistivity. Using numbers, R = 3500*10/(4*π*9) = 309 Ω.

17. If the resistors of problem 2 were replaced by 1E-6 F capacitors, what impedance would be measured across one capacitor at 1 kHz? Answer: X_c =1/(ω*C) = 1/(159*1E-6) = 6.28E3. Using a 1 mA 1 kHz AC constant current generator hooked to a node, the current splits 6 ways. Since there are no resistors or inductances, the phase relationships are unchanged and it develops, 1.04 V across each capacitor hooked to this node. Similarly a 1 mA sink does the same thing. If the source and sink are in phase and attached to each end of one capacitor 2.08V is developed for 1 mA hence X_c = 2080 Ω. This is equivalent to a 3.02E-6 F capacitor.

Chapter 10

1. Show that $x'^2+y'^2+z'^2-c^2*t'^2 = 0$ by transforming $x^2+y^2+z^2-c^2*t^2 = 0$ using the Lorentz transformation. Answer: substitute equations 10.2-5 into $x^2+y^2+z^2-c^2*t^2 = 0$ to get:
$\gamma^2*(x'^2+2*x'*t'*\beta*c+t'^2*\beta^2*c^2) + y'^2 + z'^2 - c^2*\gamma*(t'^2 - 2*t*x'*\beta*c - x'^2*\beta^2) = 0$. Canceling and grouping terms: $x'^2-x'^2*\beta^2 - c^2*t'^2+ t'^2*\beta^2*c^2 + (y'^2 + z'^2)/\gamma^2 = 0$. The first two terms are x'^2/γ^2; the third and fourth are $-c^2t'^2/\gamma^2$, hence it is proven.

2. What internationally known scientist was a high school dropout? Answer: Einstein.

3. Devise an experiment that measures directly the effect of motion on a meter stick using as a reference, an unaffected-by-the-motion meter stick. Hint: Consider the Michelson-Morley

experiment.

4. Calculate the angular distribution of radiation from an electron moving at constant velocity in a circle of constant radius. Machines designed for this purpose are called "synchrotrons" and the resulting radiation is called "synchrotron radiation." Answer: Equation 10.9-11 shows the need for $\mathscr{E}^2$. Equation 10.9-9 says $\mathscr{E}$ depends on $\underline{r}$x($\underline{r}$xv') which for the geometry shown in Figure D.10-1 is: $\underline{r}$x($\underline{r}_u$xv') = ($\underline{r}\cdot\underline{v}$')*$\underline{r}_u$ - ($\underline{r}\cdot\underline{r}_u$)*$\underline{v}$'. Here v = a*ω, v'= a*ω², $\underline{r}_u$ = $\underline{r}$-$\underline{v}$*r/c, $\underline{v}\cdot\underline{r}$ = u*r*cos(θ), $\underline{u}$'$\cdot\underline{r}$ = u*'r*sin(θ)*sin(φ). The squared triple cross product is needed which by substitution is: $\underline{r}$x($\underline{r}_u$xv') = $v'^2 * r^{4*}$. Substituting into equation 10.9-11 gives -dW/dt*dΩ = e²*v'²* {[(1-β* cos(θ)]² - [(1-β²)*sin²(θ)*cos²(φ)]}*dΩ/{(16*π²*ε_o *c^3)*[1-β*cos(θ)]^5

5. Use the concept of retarded time to show the plausibility of light production when a charged particle exceeds the velocity of light in a dielectric medium. Answer: In equation 10.9-12 retarded time enters in the denominator as 1 - n*β*cos(θ). For directions in which n*β* cos(θ)>1, the equation "blows up" and light emission takes place. A more detailed treatment using the Fourier integral is needed to estimate the energy loss.

6. What magnetic field is needed for 20 TeV (20E12 eV or 20E6 MeV) protons (rest energy = 928.3 MeV) to circle in a circumference of 56 miles? Answer: The radius is 56*5280/ (2*π* 3.28) = 14.35 km. You can use equation 10.6-9 for which γ and β are needed or you can use UCONCON on the distribution disk, I get B*ρ = 66715 T-m or a magnetic field of 4.7 T.

7. Show that the Lorentz force: *$\underline{f}$ = q*($\mathscr{E}$+ $\underline{v}$ x $\underline{B}$)* is invariant under a Lorentz transformation. Answer:. In the rest frame, consider the force from a charge at rest: *f = q*$\mathscr{E}$.* Equation 10.3-5 gives *$\underline{\mathscr{E}}$" = γ*[$\underline{\mathscr{E}}$ + $\underline{\beta}$/(c*ε_o)x $\underline{H}$]*, but $c^2 = 1/(\mu_o*\varepsilon_o)$ hence *$\underline{\mathscr{E}}$" = γ*[$\underline{\mathscr{E}}$ + $\underline{v}$ x $\underline{B}$]*. Equation 10.2-37 gives the force transformation between two frames of reference which for rest is *f = f'/γ*, and we find *$\underline{f}$ = q*($\mathscr{E}$+ $\underline{v}$ x $\underline{B}$).*

8. Elementary particles, called $\pi^\pm$s have a mean life of 2.6E-8 s, are formed in the upper atmosphere and travel toward the earth. The β of the particles is 0.9999, how far will half of them travel? Single particles decay exponentially i.e. N = N_o*exp(-t/t_m), therefore the time for half of them to decay is $t_{1/2}$ = 1.44*t_m Equation 10.4-14 is: τ/ t = √(1-β²) = 2E-4. Their half life is 2.6E-8*1.44/2E-4 = 1.87E-4 s. At about the velocity of light, half of them travel 56 km or more. Since they are formed in the upper atmosphere, they do not reach the earth's surface but it is their decay product the $\mu^\pm$ mesons that reaches the surface.

Chapter 11

1. I assumed that the angular momentum is quantized as: L = ħ*n. Planck's hypothesis was that the energy E = h*ν - not saying anything about the angular momentum. A basic test of quantum theory is "the correspondence principle" i.e. at very large quantum numbers, the results derived from quantization must approach classical results. This means that at large orbital quantum numbers, the frequency must become the frequency of the electron circling the nucleus because a classical electromagnetic radiator has the frequency of the charge circulation. Treat ħ as an undetermined constant to show, using equation 10.9-7, that in the limit of large quantum numbers, and using the classical frequency, ħ→h/(2*π).

2. If a negative mu meson (singly charged, mass = 105 MeV) is captured by a proton forming muonium (a hydrogen-like atom), what is the emission spectrum? Answer: equation 11.10-9 shows that reciprocal wavelength is: $1/\lambda = m_{e*}105/0.51*R_{d\infty}*(1/n_1^2-1/n_2^2)$.

3. Show that for elastic scattering, the relationship between the laboratory angle of the scattered projectile to the center-of-mass angle is $\sin(\theta_L-\theta_C) = m_n/A*\sin(\theta_C)$. Notice that if $m_n = A$, then $\theta_C = \theta_L/2$. Answer: Using Figure 11.13-4 and the Law of Sines $v_{nC}/\sin(\theta_L) = v_{AC}/\sin(\theta_C-\theta_L)$, but $v_{nC} = A*v_{nL}/(A+m_n)$ and $v_{AC} = m_n*v_{nL}/(A+m_n)$ substituting gives : $\sin(\theta_L-\theta_C) = m_n/A* \sin(\theta_C)$. With $m_n/A = 1$, the equation balances if substituting $\theta_C = \theta_L/2$. Hence the center-of-mass angle changes twice as fast as the laboratory angle. This limits the laboratory angle to 90° which is well known to a pool player.

4. Figure 11.14-12 shows that protons have phase stability when riding a positively increasing voltage with respect to the drift tube they are leaving. What would be the phase stable situation for H^- ions (i.e. two electrons bound to a proton). Although Figure 11.14-12 is drawn conventionally, because protons have a positive charge, they are attracted by the negative phase hence the upward ordinate in the Figure should correspond to the negative cycle of the wave, and for negatively charged particles, the Figure is correct as drawn regarding the phase stable situation and the sign of the EM wave.

5. Explain or challenge the statement, "ion betatrons are not practical." Answer: Ion betatrons are not impossible, however the alternating flux requires laminations and limitations on the choice of steel such that the typical maximum field in the doughnut is 0.1T. For a proton, the lightest of ions, at 10 MeV, $B*\rho = 0.455$T-m hence the radius is 4.55m or 29.8ft. in diameter. d<x>/dt The betatron uses the fringing field at ½ the central field to hold the particles in orbit. A ring of magnets that concentrate their field at the orbit is much more practical because they do not waste field in the center of the ring. It is much more practical to use rf cavities to accelerate the particles that the induced voltage from a changing field.

6. Prove equation 11.14-29. At the midplane $B = B_z$, therefore equation 11.14-27 is $B_z = B_o*(r_o/r)^n$. Taking logarithms and ignoring constant terms: $\ln(B_z) = -n*\ln(r)$, and taking derivatives gives $\partial B_z/\partial r = -n*B_z/r$. Rearranging $n = - (r/B_z)*\partial B_z/\partial r$

7. Carbon-14 has a half-life of 5730 years, the is formed by cosmic-ray neutron bombardment of 14N and is found in plants. The fossil specimen has an activity of 6.4 disintegrations/s*g and a modern piece of the plant has an activity of 12 disintegrations/s*g. What is the age of the specimen? Answer: Substituting into equation 11.2-2 gives: 6.4 = 12*exp(-0.6931/5730*t. Solving t = 5198 y.

Chapter 12

1. Show that the expected velocity of a particle is: $d<x>/dt = <p>/m$. Answer: The time derivative of the expected position is: $d<x>/dt = d/dt \int\psi^\star*x*\psi*d\tau = \int\psi^\star*x*(d\psi/dt)*d\tau + \int(d\psi^\star/dt)*x*\psi*d\tau$. Using Schrödinger's equation (12.2-8), $d<x>/dt = -i/(\hbar*s^2)*\int\psi^\star*x*(\nabla^2+U)\psi *d\tau$

$+i/(\hbar*s^2)*\int(\nabla^2+U)\psi^{\star}*x*\psi*d\tau = i/(\hbar*s^2)*\int[\psi^{\star}*x*\nabla^2\psi-\nabla^2\psi^{\star}*x*\psi]*d\tau$. Integrating the second integral by parts gives: $-\int\nabla^2\psi^{\star}*x*\psi*d\tau = -\int\nabla\psi^{\star}*\nabla(x*\psi)*d\tau+\int x*\psi*\nabla\psi^{\star}\cdot d\underline{S}$. The second integral which is the integral of the normal component over the surface vanishes at great distances. Performing an integration by parts on the first integral with similar vanishing at large distances giving: $-\int\nabla^2\psi^{\star}*x*\psi*d\tau = -\int\psi^{\star}*\nabla^2(x*\psi)*d\tau$. With this: $d\langle x\rangle/dt = -i/(\hbar*s^2)* \int\psi^{\star}* [x*\nabla^2\psi-\nabla^2(x*\psi)]*d\tau$ Regarding the second term: $\nabla\cdot\nabla(x*\psi) = \nabla\cdot(x*\nabla\psi+\psi*\nabla x) = \nabla x\cdot\nabla\psi + x*\nabla^2\psi = \nabla\psi\cdot\nabla x +\psi*\nabla^2 x$. But $\nabla^2 x = 0$, and $\nabla x =1$ hence $d\langle x\rangle/dt = -2*i/(\hbar*s^2)* \int\psi^{\star}*\nabla\psi/\partial*d\tau = \langle p\rangle/m$ using the replacement that $p = -i*\hbar*\nabla$ (equation 12.2-3).

2. What is the maximum time that an ice-pick can stand on its point before hitting the smooth flat surface on which it was standing? (for those of you too young to remember the ice-man, an ice-pick is a stiletto-like instrument with a rod blade tapering to a point with its mass concentrated in the handle: Answer: Assume the mass, m is at the end of length ℓ. Clearly it will stand on end longest if the distance from the vertical, Δx and its momentum Δp_x are minimum. The torque equation is $m*g*\ell*\sin(\theta) = m*\ell^2*\theta''$, where θ is the anle from the vertical. Using the small angle approximation, this is $\theta'' = \sqrt{}(g/\ell)*\theta$ which is solved by $\theta = A*\exp(\lambda*t)+ B*\exp(-\lambda*t)$, where $\lambda = \sqrt{}(\ell/g)$. When $t = 0$, $\theta = \theta_o = A+B$, and $\theta_o' = \lambda*(A-B)$. Substituting: $\theta = ½*(\theta_o +\theta_o'/\lambda)*\exp(\lambda*t)$, where the second terms is ignored because we know the angle rapidly increases with time as it falls. The Heisenberg Principle sets a limit on the product of position and momentum: $\Delta x*\Delta p_x = \hbar = \ell^2*\theta_o *m*\theta_o'$, or $\theta_o' = k*\theta_o$. Using this constraint: $\theta = ½*(\theta_o +k/(\theta_o \lambda)*\exp(\lambda*t)$. To find the maximum time, we minimize the coefficient of the exponential: $d (\theta_o +k/(\theta_o*\lambda)/d\theta_o = 0$. This occurs at $\theta_o = \sqrt{}(k/\lambda)$. The θ equation is approximately: $\theta = ½*\sqrt{}(k/\lambda)*\exp(\lambda*t)$. To find the time, the criterion of the limit of the small angle approximation $\pi/8$.; $k = 1.05E-34/(0.1*0.01) = 1.05E-31$, $\lambda = \sqrt{}(9.8/0.1) = 9.9$, and $½ *\sqrt{}(k/\lambda) = 5.1E-17$. Hence $7.7E15 = \exp(0.1*t)$ giving $t = 3.7$ s.

3. Show that eigenfunctions of the same Hermitian operator must have different eigenvalues.

4. Verify the anti-commutation relations of the Pauli spin matrices. Answer: We need to show that $\sigma_z*\sigma_x + \sigma_x*\sigma_z = 0$. Ignoring $½*\hbar$,

$\sigma_z*\sigma_x = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}*\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$. Similarly $\sigma_z*\sigma_x = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$. Adding these gives zero. The process is similar for $\sigma_z*\sigma_y + \sigma_y*\sigma_z = 0$, and $\sigma_x*\sigma_y + \sigma_y*\sigma_x = 0$

5. Derive equations 12.3-8 and 9 for L_x and L_y.

6. Show that $x^2_{n,m} = \int\psi^{\star}*x^2*\psi*d\tau = \Sigma x_{n,k}*x_{k,m}$

7. The momentum goes to zero at the boundary of a potential well. The Bessel functions, $J_{\pm1/3}(x)$ solve the Schrödinger equation with a potential $ct*(x_b-x)$ at the boundary. That is with a first order Taylor's expansion of the potential at the boundary. Connect the three solutions for: left of the well, in the well and right of the well and find the Bohr-Sommerfeld quantization relationship (equation 12.7.65).

8. Modify Computer Program 12-5 to calculate the one-dimensional hydrogen problem i.e. $U = \kappa*e^2/x$. You will have to stay away from $x = 0$.

9. Demonstrate equation 12.11-50 that by writing $\underline{p}$ in Cartesian vector form and maintaining the order of canonical conjugate variables. Take $\underline{p}$ to be only in the y-z plane.

10. Perform the multiplications of the Pauli matrices to get equations 12.3-25a,b,c.

11. Show that $\sigma_x' * \alpha_z - \alpha_z * \sigma_x' = -2*i*\alpha_y$. Hint equation 12.11-52.

12a. Prove equation 12.11-59. Hint form the dot products, use the 3 sigmas and multiply the two sums of three terms. Find the cross product as a matrix and match terms.

12b. Prove equation 12.11-65 using the suggestions for help.

12c. Prove equation 12.11-69. Suggestions: Dirac defined $\hbar^{2*}k = \beta*(2*\underline{L}\cdot\underline{s} + \hbar^2)$, square this and match up terms. It will be noted that

$$2*\underline{L}\cdot\underline{s} = \hbar*\begin{bmatrix} L_z & L_x - i*L_y \\ L_x + i*L_y & -L_z \end{bmatrix}$$

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